Minimum Cup Quantity: An Update

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Introduction

For many years there has been discussion among auditors as to what is the minimum number of catch devices (cups) needed to perform an accurate and/or acceptable audit to determine DU_{LQ} —the measure of an irrigation system's distribution uniformity. There has been difficulty in reproducing consistent results on calculated DU_{LQ} when auditing the same area by different auditors. Cup spacing, the amount of water to capture, and other test procedures are standardized by the Irrigation Association.

This paper seeks to answer the question: "is there a minimum number of catch cups required for an audit to acquire an acceptable level of approximation on DU_{LQ}"? Given an actual audit data set, does DU_{LQ} change significantly the fewer cups one uses? By creating a random exclusion simulation of real data, iterative methods of forecasting and scenario generation are used to find the error in DU_{LQ} calculation by randomly throwing out real catch cup data points and recalculating DU_{LQ}. The process is repeated 1,000 times and statistically analyzed for each catch cup quantity scenario.

Theory

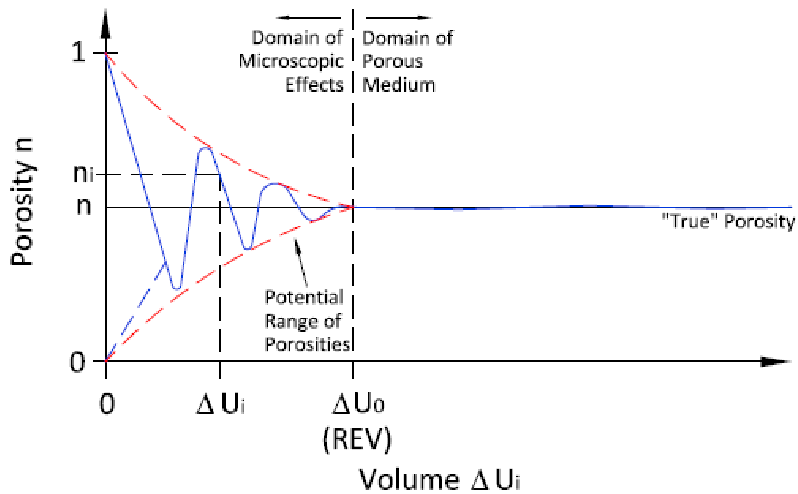
In order to determine if a minimum number of catch cups accurately describing the DU_{LQ} for an irrigation system exists, an understanding of what this represents is needed. For a given irrigation system, the DU_{LQ} is used to assess its uniformity. While there is a numerical value association (a percentage), this value is really just a qualitative estimate on how uniformly the system applies water. Audits, by definition, are just a small sample of the overall system performance. To get a "true" DU_{LQ}, an auditor would need an infinite number of catch cups covering the entire irrigation system, i.e., they would need to catch all the water. Obviously, this scenario is not feasible and auditors collect data from select areas within the irrigation system from a finite number of catch cups spaced appropriately. Therefore, DU_{LQ} is an "estimator" for system performance and is inherently variable. For example, if an auditor calculates the DU_{LQ} for an irrigation system to be 72.5, it would be acceptable for a designer to calculate uniformity and net precipitation rate from this result. One system that has a DU_{LQ} of 72.5 and another that is, for example, 67.4, in reality, have no appreciable difference in performance. The Irrigation Association (IA) provides general guidelines for distribution uniformity in that rotor systems around 80 are classified as "excellent", around 70 as "good", and closer to 55 as "poor". The goal of the audit is to ascertain whether a system is applying water uniformly or not. While quantitative data is collected during an audit, the end result should be interpreted as a qualitative assessment on system performance.

DU_{LQ} from an audit is an estimate of uniformity of water distribution for an irrigation system. If an infinite number of cups are required for the true value, and audit programs that are appropriate in size and coverage can closely approximate this value, then there exists a relationship between the number of catch cups and the representative DU_{LQ} value. Therefore, if for a given audit one increases the number of catch cups to better approximate the actual value, is the converse true? In other words, if one reduces the number of catch cups, does the DU_{LQ} deviate further from the real DU_{LQ}? Experience, and perhaps intuition, tells us that the answer to these questions is yes. But, DU_{LQ} from an audit is not required to be an exact or perfect representation of the truth: it is intended to be an estimate. So, we arrive at the original question laid out more specifically: is there a minimum number of cups required for an audit to acquire an acceptable level of approximation on DU_{LQ}?

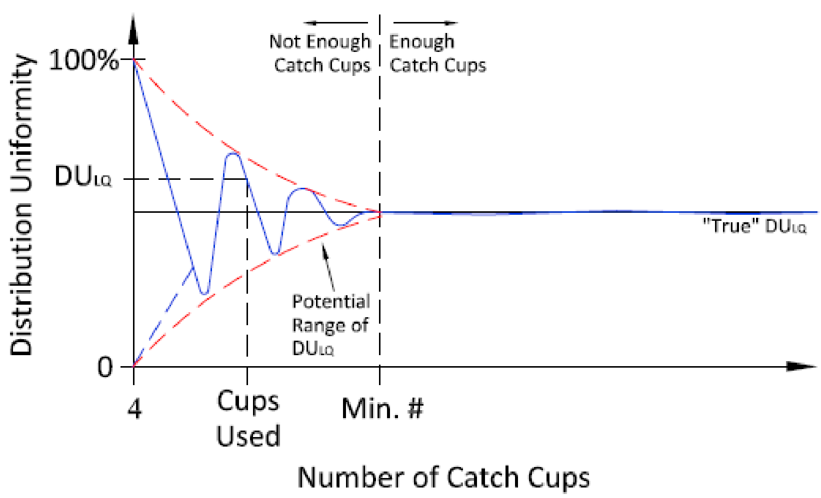
This question has been asked in other fields of science and engineering for different index parameters and estimators. Bear (1972) provides a robust discussion on the continuum approach and the concept of a Representative Elemental Volume (REV) for soil properties such as porosity as shown in Figure 1a. For example in production well design, if one could perform testing on the entire aquifer volume, the true porosity, n, would be known. Yet, a very good approximation (n_i) could be made if only 10 cubic yards of aquifer material were tested, or one cubic yard, and so on (each represented by some volume ΔU_i). Testing for porosity is generally limited to a few split-spoon samplers—the total volume of which would be, at most, on the order of one cubic foot. However, for example, if one split-spoon sample, or a cup, or a teaspoon of aquifer material is taken for testing, then the variations due to sample location, grain-size distribution, etc., can skew the resulting porosity values. "Microscopic effects" would begin to provide results that would not be "representative" values of the true porosity. In the examples described by Bear, there exists a minimum volume that can still represent the true value of the desired parameter (ΔU_0). This concept is crucial to testing design and construction management as minimizing testing minimizes costs. The same concept can be applied to irrigation auditing in determining a minimum number of cups to represent the entire system. The following research verifies the behavior of audits correlated to a REV in Figure 1b.

Analysis

The IA provides guidelines for auditing procedures in that cup spacing is somewhat standardized—relative to the spacing of the irrigation heads. Generally, cups are placed at regular and appropriate intervals at and in between heads. Cup spacing and the number of cups are related to one another when considering a fixed audited area such as a golf green or lawn. As it will be further explained below, this analysis takes actual audit data, randomly removes some of the cup data, simulates a new audit, and notes the effects on the calculated DU_{LQ}. It is assumed that the simulated audit (with less data points) have catch cups that are evenly spaced between each other. So, when data are removed from the original audit, we are, technically, changing the spacing, as the catch cups will spaced further apart. However, the simulated audits as a whole will still make an assessment on the entire area in question. This paper is not a study on cup spacing, per se: it is a study on the level of discretization required to properly calculate DU_{LQ}. As more catch cup data points are taken away, the spacing would be one cause for uncharacteristic uniformity calculations. The authors recognize that audits of rotary and fixed-arc spray sprinkler heads may differ and have included both types in the following analysis.



a) Representative Elemental Volume (ΔU_0) for Porosity



b) REV Concept for Irrigation Audit Catch Cups

Figure 1: The Representative Elemental Volume Concept (from Bear, 1972)

Actual Data DU_{LQ} Analysis		Random Exculsion Simulation Analysis	
Cup Count	**36**	*Cup Count*	**32**
Cups in LQ	**9**	*Cups in LQ*	**2**
Total Collection (mL)	**1497**	*Total Collection (mL)*	**287**
LQ Collection (mL)	**149**	*LQ Collection (mL)*	**25**
Average Total	**41.58**	*Average Total*	**35.88**
Average LQ	**16.56**	*Average LQ*	**12.50**
Actual DU_{LQ}	**39.81%**	*Random Data DU_{LQ}*	**34.84%**

a) Sample RES Program Simulation Output for an Actual Audit

Number of Catch Cups used from Actual	4	8	12	16	20	24	28	32	36
Actual Field DU_{LQ}	39.8%	39.8%	39.8%	39.8%	39.8%	39.8%	39.8%	39.8%	39.8%
Average Distribution Uniformity, DU_{LQ}	49.2%	43.4%	42.3%	41.0%	40.8%	39.9%	40.0%	39.8%	39.8%
Standard Deviation, s, of DU_{LQ} Sampling	18.1%	10.8%	7.9%	6.3%	5.2%	3.9%	3.1%	2.0%	0.0%
Minus 1 Standard Deviation	31.1%	32.6%	34.3%	34.7%	35.5%	36.1%	36.9%	37.8%	39.8%
Plus 1 Standard Deviation	67.3%	54.2%	50.2%	47.3%	46.0%	43.8%	43.2%	41.8%	39.8%
% Difference of Sampling DU to Actual DU	23.5%	9.0%	6.2%	3.0%	2.4%	0.3%	0.6%	0.1%	0.0%
Maximum Random Sampling DU_{LQ}	97.6%	90.3%	79.1%	61.6%	57.3%	53.1%	50.8%	47.6%	39.8%
Minimum Random Sampling DU_{LQ}	17.2%	21.3%	25.3%	26.0%	28.5%	31.3%	33.0%	36.0%	39.8%
Difference Between Max and Min DU_{LQ}	80.4%	69.0%	53.8%	35.6%	28.8%	21.8%	17.8%	11.6%	0.0%
Simulation Iteration	4	8	12	16	20	24	28	32	36
1	90.9%	27.2%	50.9%	32.8%	52.2%	42.6%	36.3%	38.7%	39.8%
2	58.8%	34.6%	42.2%	50.5%	38.8%	33.1%	42.6%	41.7%	39.8%
3	47.1%	44.4%	40.6%	47.4%	39.1%	39.5%	41.0%	42.7%	39.8%
4	43.9%	41.4%	50.0%	41.6%	42.4%	31.7%	39.7%	38.1%	39.8%
5	44.9%	48.6%	40.3%	33.9%	32.7%	41.3%	42.3%	39.3%	39.8%
6	46.2%	36.5%	55.1%	44.8%	34.4%	37.7%	36.0%	41.3%	39.8%
7	64.5%	31.1%	41.8%	35.9%	35.9%	38.7%	37.7%	38.5%	39.8%
8	35.7%	27.7%	49.6%	38.5%	36.5%	43.9%	37.5%	37.8%	39.8%
9	40.7%	54.3%	40.4%	55.2%	53.0%	38.6%	40.3%	40.2%	39.8%
10	20.7%	23.0%	28.9%	45.2%	46.5%	32.5%	41.8%	41.4%	39.8%
995	55.6%	30.2%	49.9%	29.2%	44.2%	36.9%	36.1%	38.4%	39.8%
996	24.5%	29.4%	40.4%	35.4%	34.8%	39.9%	37.6%	39.5%	39.8%
997	60.2%	44.2%	35.3%	50.0%	41.3%	36.9%	43.8%	37.7%	39.8%
998	31.0%	47.1%	49.6%	40.4%	48.4%	33.9%	38.0%	42.3%	39.8%
999	77.3%	45.7%	34.6%	42.6%	48.4%	32.9%	34.9%	39.6%	39.8%
1000	55.2%	69.3%	32.1%	48.6%	31.8%	35.3%	39.8%	42.2%	39.8%

b) Abbreviated Output of 1,000 Simulations of RES Data Sorted and Analyzed

Figure 2: Random Exclusion Simulation Analysis for Audits

Using the random number generator in Excel, a method was developed to take actual audit data, randomly remove a user-specified number of catch cups, and recalculate the new DU_{LQ}. A simulation was run for a specific number of catch cups 1,000 times each. For example, if an original audit had 40 catch cups, then 1,000 simulations were performed for a 36-cup audit, 1,000 for a 32-cup audit, etc. The average, minimum, maximum, and variance of the DU_{LQ} for the 1,000 simulations were calculated. These were compared to the actual distribution uniformity that was calculated in the effort to show the progressive degradation of reliability of using too few cups. The computer analysis performed is a Random Exclusion Simulation (RES) of real data to acquire a new DU_{LQ} calculation. Figure 2 displays excerpts from the program's interface to show how the analyses were executed. Note that these analyses adhere to IA guidelines for DU_{LQ} calculation by only using audits with a total catch cup count as a multiple of four. This way (as the IA intended it) when dividing data into quartiles, they are evenly grouped and no interpolation is required. The method of taking real data and generating simulated scenarios is not new in analytical studies. This "re-sampling" analysis method is almost identical to bootstrapping—a common statistical practice. However, bootstrapping requires that the re-sampled size be identical to the actual size. By using actual data in the simulations, we are utilizing the same distribution of catch cup volumes found in the field to apply it to a simulated run—giving a sense of reality in the synthesized audit and validating the analysis. A sensitivity study as to the number of simulations to run was also carried out. RES program runs for 5,000 and 32,000 (the maximum Excel could handle) simulations were performed. It was determined that there was no appreciable difference in the number of iterations used between 1,000 and beyond. The computing time was about 10 times greater with 5,000 simulations (10 minutes) while 32,000 simulations took over an hour (when the PC used didn't crash). Therefore, the accuracy level and computing time were found acceptable for 1,000 simulations.

Results

Figure 2b shows an abbreviated output for a typical RES program run. 1,000 simulations were performed for each new catch cup total. The actual audit data removes no cups in the simulation and, therefore, retains its actual data in the simulation (39.8 DU_{LQ} in all simulations). As the number of catch cups is reduced to 32, 28, etc., the DU_{LQ} changes for each simulation. The average DU_{LQ} for the each new catch cup count begins to slightly deviate from the actual field result by trending higher. What becomes noteworthy in the example shown in Figure 2 is that while the simulated DU_{LQ} stays very close to the actual value, the standard deviation of the simulated average grows larger. Moreover, the minimum and maximum ranges DU_{LQ} become larger by decreasing the number of catch cups. Graphically, these phenomena are shown in Figure 3.

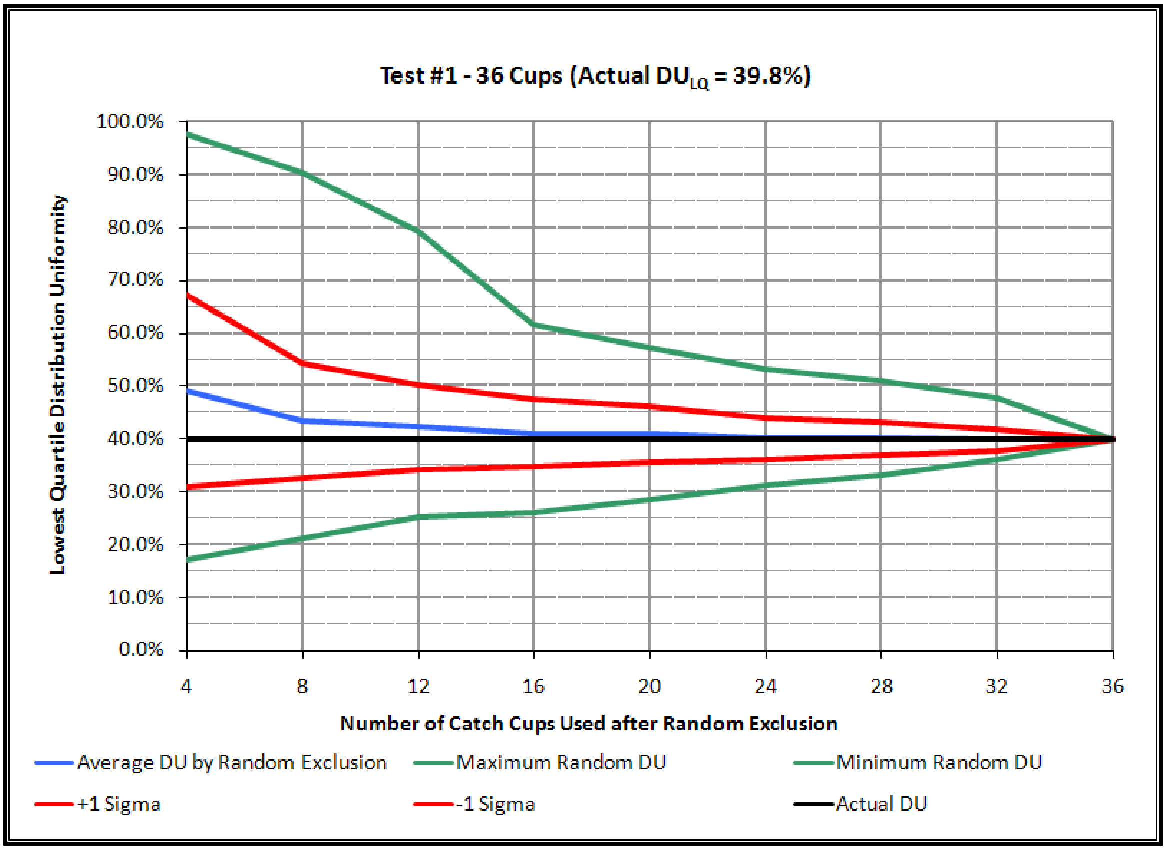


Figure 3: Graphical Results of Sample RES Program Output for an Actual Audit

The actual DU_{LQ} for this test (black line) is assumed to be the "true" DU_{LQ} of the system. If one reduces the amount of catch cups in an audit, on average, one should calculate almost exactly the same DU_{LQ}. However, the variations in results become greater, as shown by the standard deviations (red lines) and extreme values (green lines). If another auditor goes out to this site and re-tests the system with, for example, 20 cups, the probability becomes greater that he or she will come up with a DU_{LQ} that is not representative of actual conditions. The effects of low discretization levels become evident as fewer cups are used similar to the "microscopic effects" on porosity in Figure 1a.

To illustrate how the issue of using too few catch cups becomes a probability issue, consider Figure 4. This chart tabulates the number of occurrences out of 1,000 simulations that the simulated DU_{LQ} falls within a bandwidth of 2.5. Note that the bar graph for 36 cups, the number of cups used in the actual field audit (DU_{LQ} = 39.8), have 1,000 occurrences that fall between 37.5 and 40.0. The black vertical lines indicate the +/- 5 range on actual DU_{LQ} (34.8 - 44.8). The results of each catch cup scenario run 1,000 times are overlaid on each other to show the distribution of occurrences. The key visual aspects to understand from this figure are that when there are a sufficient number of cups in the simulated audits, the average DU_{LQ} of all iterations stays close to the actual DU_{LQ} and that the band of DU_{LQ} calculated is narrower and stays symmetrical around the actual DU_{LQ} value. On the other hand, when there are an insufficient number of cups used in simulation, the average DU_{LQ} trends away from the actual value, the distribution loses symmetry about the field DU_{LQ}, and the values are scattered about all ranges in a wide band of possible values. Based on the tabulated data, if, for example, one were to perform a new audit for the area analyzed and spaced out only 20 cups for the test, there is only 75% chance that the DU_{LQ} obtained will be between the +/- 5 range on actual DU_{LQ}. The probability drops to 43% when that number drops to 8 cups. However, this chance increases to 99% when 32 cups are used. Therefore, if the number of cups used in an audit goes down, there is a greater the chance that the audit will provide non-representative results.

This type of analysis with trends and probabilities on simulated DU_{LQ} calculation was performed using 13 actual audits on large spaced rotor sprinkler systems. These audits range from 32 – 128 cups and took place in locations all over the contiguous United States. The resulting data were combined in order to ascertain the key parameters in determining a minimum number of catch cups to accurately represent distribution uniformity. Five parameters were calculated from the combined analyses and shown in Figure 5:

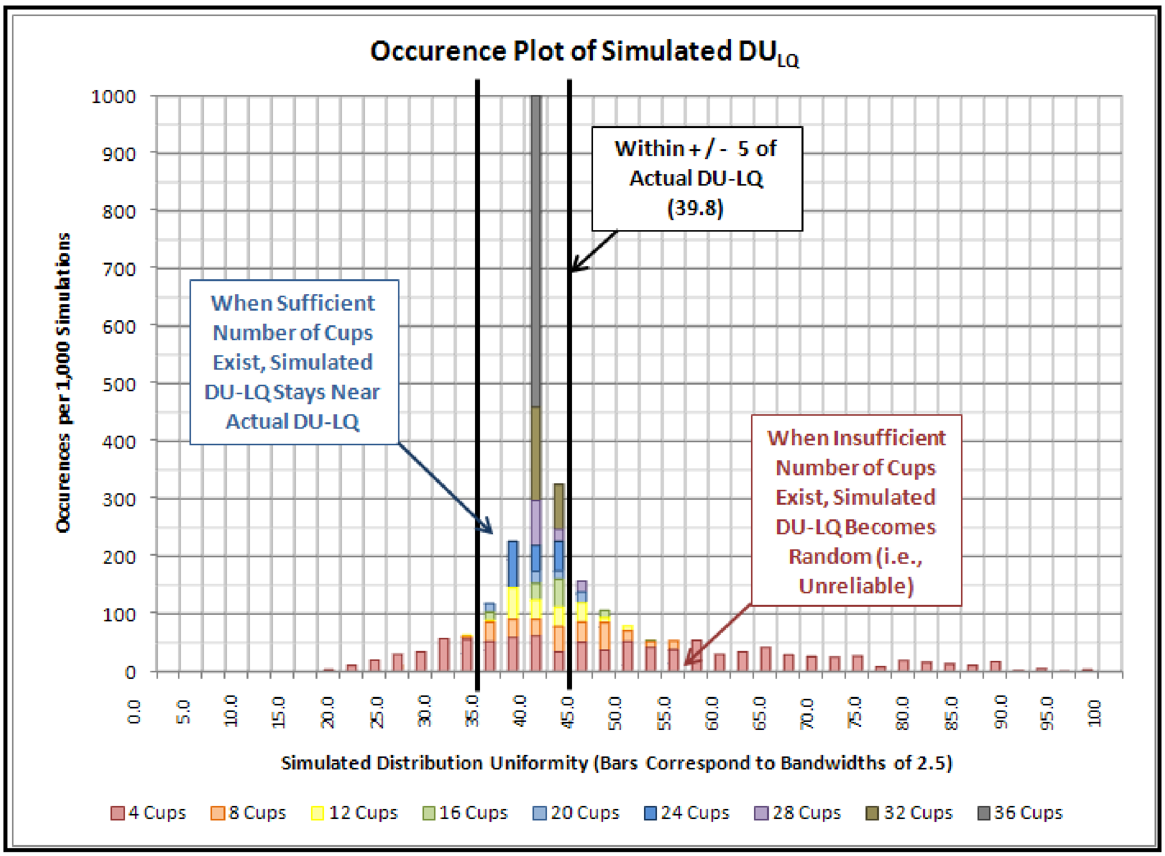


Figure 4: Number of Occurrences DU_{LQ} falls within bandwidths of 2.5 in 1,000 iterations of Random Exclusion Simulation data (Actual Field Audit has 36 Cups)

1.) Average Actual DU_{LQ} = 46
2.) Average on DU_{LQ} Simulations
3.) Change of Average Simulated DU_{LQ} to Average Actual DU_{LQ}
4.) Average Standard Deviation on DU_{LQ} Simulations
5.) Average Number of Occurrences (out of 1,000) Simulated DU_{LQ} is within +/-5 of Actual DU_{LQ}

Average Field DU_{LQ} = 46.3	Number of Catch Cups in Random Exclusion Simulations (All Data Analyzed)									
	4	8	12	16	20	24	28	32	36	40
Average Simulated DU_{LQ}	56.4	51.2	49.1	48.0	47.6	47.3	47.0	46.8	46.7	46.6
Difference Between Average and Simulated DU_{LQ} From Field DU_{LQ}	10.1	4.9	2.8	1.7	1.3	1.0	0.7	0.5	0.4	0.3
Standard Deviation of Average Simulated DU_{LQ}	18.5	12.1	9.2	7.4	6.1	5.0	4.2	3.2	2.5	1.5
Occurrences per 1,000 Simulations of DU_{LQ} Within +/-5 of Field DU_{LQ}	218	378	502	599	695	776	852	912	952	965

Figure 5: Combined Statistical Data on Large Spaced Rotor Sprinkler Audits

These calculated parameters were selected because it allows the experienced auditor to make some global sense on the sensitivity on catch cup totals. In trying to sift through this data and make some assertions, design judgment is used to try to narrow down a minimum number of catch cups to use. The criteria set forth for group averages were:

1.) The difference in average simulated DU_{LQ} is within +/- 1.25 of the actual DU_{LQ}. This would place the simulated DU within the bandwidth of 2.5 encompassing the actual DU in the Occurrence Plot of Simulations shown in Figure 4.
2.) A standard deviation of 5 in simulated DU_{LQ}. There is not much difference *qualitatively* between a DU_{LQ} of 41 and 51. A range of 10 is a sensible estimator for comparing audits.
3.) Greater than 75% of all occurrences (out of 1,000) are within +/- 5 of the actual DU_{LQ}.

These criteria should not be considered completely arbitrary. Given the analysis the authors have described, the IA guidelines for audits, and auditing experience, they are a feasible starting point to consider a minimum number. The number of catch cups where the averages for all 13 audits with large rotor sprinklers meet all the given criteria is 24.

When examining medium rotor sprinklers and applying the same criteria previously described, a similar result is found when averaging 5 audits. Figure 6 replicates the summary in Figure 5, except for medium throw rotors.

Average Field DU_{LQ} = 48.7	Number of Catch Cups in Random Exclusion Simulations (All Data Analyzed)									
	4	8	12	16	20	24	28	32	36	40
Average Simulated DU_{LQ}	57.3	53.2	51.6	50.6	50.0	49.6	49.3	49.2	49.0	48.9
Difference Between Average and Simulated DU_{LQ} From Field DU_{LQ}	8.6	4.5	2.9	1.9	1.3	0.9	0.6	0.5	0.3	0.2
Standard Deviation of Average Simulated DU_{LQ}	17.9	11.4	8.9	7.0	5.7	4.7	3.7	2.4	3.0	1.4
Occurrences per 1,000 Simulations of DU_{LQ} Within +/-5 of Field DU_{LQ}	245	412	543	647	742	817	885	924	924	958

Figure 6: Combined Statistical Data on Medium Spaced Rotor Sprinkler Audits

With the analysis above, again 24 cups is the minimum number of cups where all criteria are met. To this point, it would appear that initial spacing for audits would have very little effect on the minimum number of catch cans. In both the large and medium rotary sprinkler analyses, more audit data should be included to statistically strengthen these assertions. Nonetheless, Figures 5 and 6 show that when fewer cups are used in an audit, the calculated DU_{LQ} diverges from the true DU_{LQ}. These analyses attempt to pinpoint a minimum number of audit catch cups that minimize the amount of acceptable divergence from a representative value, i.e., to minimize error.

Spray sprinklers are generally much closer and in greater numbers within an irrigation system compared to rotors. Unlike rotary sprinkler audits where all sprinklers are operational at the same time over a given audit area, spray sprinkler audits generally require that individual zones are run sequentially due to flow restrictions and/or water availability. The same analysis above was applied to 6 spray zone audits ranging from 192 – 600 cups on outdoor testing facilities.

Average Field DU_{LQ} =64.3	Number of Catch Cups in Random Exclusion Simulations (All Data Analyzed)									
	4	8	12	16	20	24	28	32	36	40
Average Simulated DU_{LQ}	70.5	67.5	66.5	66.0	65.6	65.3	65.1	65.0	64.9	64.7
Difference Between Average and Simulated DU_{LQ} From Field DU_{LQ}	6.2	3.2	2.2	1.7	1.3	1.0	0.8	0.7	0.6	0.4
Standard Deviation of Average Simulated DU_{LQ}	13.7	9.7	7.9	6.9	6.1	5.6	5.1	4.8	4.4	4.2
Occurrences per 1,000 Simulations of DU_{LQ} Within +/-5 of Field DU_{LQ}	295	452	547	608	677	714	758	778	797	813

Figure 7: Combined Statistical Data on Spray Sprinkler Audits

When applying the same criteria to spray audits as in the rotor audits, only one of the 3 criteria is met at 24 catch cups (difference between simulated and average). The standard deviation criteria is met at 32 catch cups, while 750 occurrences or greater per 1,000 is met at 28 catch cups. While the desired result of 24 cups as a minimum to meet all criteria was not realized for spray data alone, at least as importantly, the same trends of increasing divergence from actual DU_{LQ} values with fewer catch cups remain intact.

Conclusions

In all analyses, the authors recognize that more audits are required to make a definitive answer as to what the minimum number of catch cups for an audit should be. The results presented above begin to point strongly towards 24 as a minimum number. The criteria for passing acceptability are based on total number of catch cups was synthesized from experience, design judgment, and basic statistics. For a more robust statistical analysis, involving confidence intervals, exceedance probabilities, etc., more audits would be required.

Average Field DU_{LQ} = 51.3	Number of Catch Cups in Random Exclusion Simulations (All Data Analyzed)									
	4	8	12	16	20	24	28	32	36	40
Average Simulated DU_{LQ}	60.1	55.7	54.0	53.0	52.6	52.3	52.0	51.9	51.7	51.6
Difference Between Average and Simulated DU_{LQ} From Field DU_{LQ}	8.8	4.4	2.7	1.7	1.3	1.0	0.7	0.6	0.4	0.3
Standard Deviation of Average Simulated DU_{LQ}	17.2	11.4	8.8	7.2	6.0	5.088	4.3	3.4	3.1	2.2
Occurrences per 1,000 Simulations of DU_{LQ} Within +/-5 of Field DU_{LQ}	243	404	522	611	700	769	835	881	907	926

Figure 8: Combined Statistical Data on All Audits

In essence, a prudent auditor should have in mind what their minimum level of error and variation is. Using the data presented above, it may be evident that “somewhere between 24 and 32 cups” would be acceptable. It would have been desirable to have the minimum number be 24 in each independent audit analysis based on sprinkler type. However, if a minimum number had to be selected from the data above, as a final analysis, all audits (coincidentally, 24 total) are averaged together and tested against the criteria described above.

Figure 8 displays the results of all audits averaged together regardless of sprinkler type. If we are to adhere strictly to the criteria for acceptable error presented above, then the difference between simulated and field DU_{LQ} would be met at 24 cups, the number of occurrences criterion would be met at 24 cups, but the standard deviation of simulated DU_{LQ} would be met at 28 cups. However, since the criteria presented above are based mostly on experience and judgment, both would indicate to the rational auditor that at a standard deviation of 5.088, 24 catch cups could be within the realm acceptability for error in irrigation auditing.

More catch cups in an audit lead to less error and a more realistic sense as to the “true” DU_{LQ} of an irrigation system. However, based on the data and ideas presented above, with time and money as constraints for set up, testing, and analysis of audits, the minimum number of catch cups to have the best chance of finding a “representative” value for DU_{LQ}, and thereby ascertaining the “true” efficiency of an irrigation system, is 24.