The Use of Low-Cost, Differentially-Corrected GPS for Reporting Field Position of Self-Propelled Irrigation Systems

R. Troy Peters\textsuperscript{2}, Dale F. Heermann\textsuperscript{3}, Kristine M. Stahl\textsuperscript{3}

Abstract

Precision irrigation and chemigation using center pivots and lateral move systems requires precise knowledge of the field position of the moving irrigation system. This kind of precision is not possible with typical methods of reporting angular position of center pivots. Lateral move systems do not have a readily-available, low-cost method of reporting field position. The decreasing costs and increased precision of differentially corrected GPS receivers make them a possible solution to this problem. Low-cost GPS units were tested at stationary positions in Bushland Texas and in Fort Collins, Colorado. Tests on a moving center pivot were performed in Fort Collins. Outlying errors from the reported GPS positions can be mitigated by averaging the GPS positions. Two different averaging methods were evaluated and an algorithm for combining these averaging methods with dead reckoning for reporting real-time position using recent historical data is presented. Various time periods for averaging GPS points were evaluated. Averaging time periods from 10 to 30 minutes bring in the outlying errors sufficiently without having to apply dead reckoning across long times and distances. There was good agreement on the moving pivot between GPS calculated angular data when compared with measured reference points. The position estimates improved by averaging over greater time periods. Averaging GPS points to calculate angular velocity decreases the variability of velocity estimates. Averaging times of 5 to 10 minutes appeared adequate to give good estimates of angular velocity.

Introduction

Along with increased accuracy, the cost of differentially corrected GPS receivers has been decreasing, making possible their use in many additional applications. One such application includes precision farming where GPS systems are used to guide tractors, and collect position and yield data to create yield maps. There has been additional interest in using GPS technology for center-pivot or lateral-move positioning with precision or site-specific irrigation and/or chemigation.

\textsuperscript{1} Contribution from USDA-Agricultural Research Service, Conservation and Production Research, Bushland, TX and Water Management Research, Fort Collins, CO. The mention of trade or manufacturer names is made for information only and does not imply an endorsement, recommendation or exclusion by USDA – Agricultural Research Service.

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Most modern center pivots use either a resolver or an optical encoder located at the pivot point to report angular position. However, these are often subject to errors and can only report the position of the first tower from the center point. Since the pivot end tower on an electrical drive pivot typically moves much more frequently than the first tower and there may be a bow in the pivot alignment, the reported angular position of the first tower may not translate into an accurate representation of the position of the end tower. Other site-specific irrigation research has found errors in the pivot position angle reported by the control panel and identified correction algorithms to get accurate field positions (e.g. Sadler et al., 2002; Peters and Evett, 2005a). Though these position errors are not a cause for concern for most irrigators, accurate, real-time knowledge of pivot position is required for site-specific irrigation. A low cost GPS receiver mounted near the end of the pivot has the potential to provide a more accurate representation of the pivot’s position (Peters and Evett, 2005a). Most lateral-move control systems do not have an accurate, low-cost mechanism for reporting field position. Heermann et al. (1997) discussed the position reporting alternatives and concluded that GPS was the most viable method for determining field position for lateral-move systems.

Heermann et al. (1997) investigated non-differentially corrected GPS positioning on a lateral-move irrigation system for site-specific irrigation work. They determined potential position with dead reckoning based on travel speed and known initial position. This was then corrected with an averaging algorithm applied to the GPS receiver reported positions. The demonstrated accuracy was within plus or minus 7 m. Kostrzewski et al. (2002) briefly described a lateral-move system with a differentially corrected GPS unit mounted on one end for reporting system position. In this experiment the position accuracy was described by fitting a regression curve to the measured points from a moving system and the variance from the regression was discussed. Reinke Manufacturing Inc. (Deshler, Nebraska) has applied for a patent (Barker, 2004) for a GPS control system for mechanized irrigation systems. GPS units are being tested on cornering systems (Robinson, 2003). Peters and Evett (2005a) investigated the accuracy of low-cost GPS units as applied to center pivots or lateral-move irrigation systems and found that significant improvement of angular position reporting was possible. The tested low-cost receiver was accurate to within 2.1 m 95% of the time. However, the remaining 5% of points had errors as large as 6.6 m (Peters and Evett; 2005a). Peters and Evett (2005b) also investigated the use of a second, similar GPS receiver in a known location (like the pivot center point) to correct for the errors of the moving receiver (mounted on the pivot end point) and discovered that there was very little correlation between the errors of the two receivers. Although GPS has much better accuracy than traditional methods of reporting field position of self-propelled irrigation systems, their application for center pivot or lateral move positioning is hindered by large outlying errors and the fact that the reported position tends to fluctuate in time. These errors can be mitigated by using average position locations instead of single, real-time-reported locations from the receiver. The objective of this research is to evaluate the effectiveness of averaging algorithms for controlling these outlying errors and to present methods of using averaged positions with dead reckoning for more precise position estimates of moving, self-propelled irrigation systems.
Materials and Methods

Bushland, TX Data Collection
A low-cost, differentially-corrected GPS receiver (Garmin 16HVS; Garmin International; Olathe, KS), was mounted past the end tower on a center pivot with a 127 m (417 ft) radius located at the Conservation and Production Research Laboratory of the USDA, Agricultural Research Service (ARS) in Bushland, Texas. The GPS receiver was wired into a Campbell Scientific datalogger (CR10X). The datalogger recorded data from the GPS output NMEA (National Marine Electronics Association) sentence ($GPGGA$), which used the RS-232 protocol, every second and logged the one minute averages of the sensed location. The pivot was left in a stationary location for three different extended time periods of five days or greater on days of year (DOY) 194-199 (132 hours), 215-221 (131 hours), and 224-232 (183 hours). Since the sensors were not on a known benchmark, precision was calculated instead of accuracy and the relative position to the mean was evaluated.

The reported positions in longitude and latitude were translated into X-Y positions on a theoretical grid using a series of equations described by Carlson (1999). These equations used the WGS-84 (World Geodetic Survey 1984) reference datum to determine the earth’s spheroid model. The average position of the receiver was set as the axis origin and the variations of the individual measurements from the mean were calculated. The pivot’s known location is used with the reported end location to calculate the center pivot’s angular position.

Fort Collins, CO Data Collection
A similar, low-cost, differentially-corrected GPS receiver (Garmin GPS 17N; Garmin International; Olathe, KS) was also mounted both at a stationary position and one foot beyond the center pivot end tower with radius of 79 m (260 ft) located at the Agricultural Engineering Research Center, Colorado State University, Fort Collins, Colorado. The data were collected on a laptop computer at three second intervals. The processing was similar to the procedure at Bushland above. A long term data set (approximately 68 hours) was collected at a stationary position. Data were also collected from the moving center pivot with the percent timer setting of 50 and 100%. Seven reference stakes were placed just outside of the outer tower and the times of the pivot passing each stake were recorded. The moving data were analyzed in terms of errors in both radius and angle of rotation. The angle, $\theta$, is the only unknown since the radius is a constant when mounted on a pivot. However, since the radius is known, the error in measuring this length with the GPS provides an insight as to the accuracy expected when the pivot is stationary or moving.

Averaging Algorithms and Dead Reckoning
Peters and Evett (2005a; 2005b) and others have shown that although differentially corrected GPS is quite good at providing an accurate estimate of position, the observed large outlying errors are a cause for concern for applications in precision irrigation. It was hypothesized that averaging the reported positions over various time intervals could reduce these errors. Determining the real-time position using averaged sensed GPS
positions on a moving center pivot requires a method of estimating the current position from past positions. Development of an algorithm combining dead reckoning and GPS measurements is possible.

Dead reckoning is defined as the use of a known beginning point, velocity and direction of travel to determine the ending location. When used on a pivot, the beginning reference position is the latest averaged GPS position. Average GPS positions can be determined in two ways. The first is to take the average position during specific time intervals such as between every five minute mark. This is termed a time-period average. The other is a rolling average. With a rolling average, all reported locations within a specified time interval are held in memory. Each new point is included in the average as it comes in and the oldest point is excluded such that the number of averaged points remains constant. Both methods (time-period, and rolling averages) can be taken over various time intervals. The time intervals of 3 seconds data, and 1, 5, 10, 30 and 60 minutes averages were tested to determine which would be most ideal for accurate center pivot position reporting. An advantage of rolling averages is that dead reckoning is applied across a time only half as long as the chosen time interval for averaging. For example if a rolling average was taken on a 60 minute time interval then dead reckoning must be applied from the average point forwards 30 minutes. However, with time-period averages dead reckoning must be applied across longer times from 0.5 to 1.5 times the averaging interval. For example if a 60 minute time interval is used then at the extreme, dead reckoning must be applied across a 90 minute time interval before the next 60 minute average is updated. Applying dead reckoning across longer times and distances may be an added source of uncertainty. Taking rolling averages has the distinct disadvantage, however, of requiring the retention of all of the data points and their order in memory so that the oldest point may be dropped when the newest point is included in the average. This may complicate programming and increase memory requirements considerably.

The other unknowns for dead reckoning are travel speed and direction. Pivot travel speed can be determined from the commonly known time that the pivot takes to make a complete 360 degree revolution ($t_{rev}$) at the 100% setting, or traveling as fast as possible. The travel speed in degrees/minute can be calculated using this time as:

$$\Delta \theta / \Delta t = \frac{Pcnt}{100} \cdot \frac{360}{t_{rev}}$$

where $\Delta \theta / \Delta t$ is the travel speed in degrees per minute, and $Pcnt$ is the pivot timer’s percent setting which is available from the pivot’s electronic control panel. This method of calculating $\Delta \theta / \Delta t$ will not be able to identify slippage or unplanned changes in velocity. $\Delta \theta / \Delta t$ can also be calculated using GPS data from the recent past. This is as simple as choosing a time interval ($\Delta t$) and measuring the change in angular position over this time interval ($\Delta \theta$) and calculating $\Delta \theta / \Delta t$. The analysis of error with different averaging times would provide an estimate of any changes in $\Delta \theta / \Delta t$ for calculating the angle $\theta$.  

275
Averaged GPS positions from a moving center pivot or lateral move can be combined with dead reckoning using *time-period averages* as:

\[
X_{\text{new}} = \bar{X} + \left( \frac{t}{2} + t_{\text{now}} - t_{\text{LastEnd}} \right) \Delta \theta / \Delta t \tag{2}
\]

where \(X_{\text{new}}\) is the new real-time angular position of the pivot, \(\bar{X}\) is the averaged GPS position, and \(t\) is the time period over which the GPS position is averaged in minutes, \(t_{\text{now}}\) is the current time, and \(t_{\text{LastEnd}}\) is the time that the last time-period average was updated. Position can be reported as an angular position for center pivots, or a position from starting point for lateral move systems. The same calculation using *rolling averages* is computed as:

\[
X_{\text{new}} \approx \bar{X} + \frac{t}{2} \Delta \theta / \Delta t . \tag{3}
\]

### Results and Discussion

The overall results from using the one-minute averages for the three different trials at Bushland are given in Figure 1. Although most position estimates are within 1 – 2 meters of the mean, there are a few outlying points that go far beyond this.

![Figure 1](image.png)

Figure 1. Cumulative probability that the error is less than the given distance from the mean for the three different extended time periods.
The one minute average position for the DOY 194-199 time period is plotted in Figure 2. This shows the large amount of variability, and especially the outlying points. To show the effect of averaging, the 10 minute and 60 minute average positions (time-period averages) for the same time period are plotted in Figures 3 and 4. The effect that averaging has on bringing in the outlying points is dramatic.

Although it is clear that averaging bring in outlying points, they do not have much effect on the average deviation from the mean. Table 1 gives the descriptive statistics for the errors (distance from the mean) of the three Bushland trials using time-period averages. Table 2 gives the same statistics for the errors of the stationary trial in Fort Collins. Although the maximum error is reigned in significantly and the root mean squared error is decreased slightly, the other error terms are only slightly affected. The 50th and 90th percentile of the distribution sometimes actually increase when more points are included in the average. This may be due to the fact that some errors are not random, but effected by things which change slowly over time such as the atmospheric influences on the signal speed from the satellites. It is not clear why the errors from the data collected on DOY 194-199 are so much higher than the other days.

For comparison rolling averages using the same time intervals were taken with the same data from Bushland (Table 3). The errors of these rolling averages for the same time interval are generally larger than for the time-period averages (Tables 1 and 2). Again this may be due to the slowly changing atmospheric conditions which would cause the errors to follow each other around.

Figure 2. 1 minute averages for the DOY 194-199 time period.
Figure 3. 10 minute averages for the DOY 194-199 time period.

Figure 4. 60 minute averages for the DOY 194-199 time period.
Table 1. Time-period averages of three different extended data collection periods in a stationary location in 2005 from DOYs 194-199, 215-221, and 224-232 in Bushland, TX. The averages were taken on 1, 5, 10, 30, and 60-minute time intervals. Statistics on the error (distance from the mean, m) include the mean, the root mean square error (RMSE), the 50% (median) and 95% distribution, the maximum error, the standard deviation, and the number of points included (N).

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Table 2. Time-period averages of a 68 hour data collection period in a stationary location for 2002, DOY 262-265 in Fort Collins, CO. The data were collected on 3 second intervals, and averages were taken on 1, 5, 10, 30 and 60-minute timer intervals.

Statistics on the error (distance from the mean, m) include the mean, the root mean square error (RMSE), the 50% (median) and the 95% distribution points, the maximum error, the standard deviation, and the number of points included (N).

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<td>6.08</td>
<td>4.25</td>
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<tr>
<td>StDev</td>
<td>1.57</td>
<td>1.12</td>
<td>0.98</td>
<td>0.80</td>
<td>0.70</td>
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<td>N</td>
<td>7897</td>
<td>7893</td>
<td>7888</td>
<td>7868</td>
<td>7838</td>
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A graphical representation of the maximum error and the root mean squared error data for the different trials are given in Figures 5 and 6. Again, the advantages of averaging are clear. Figures 5 and 6 also show that the differences between time-period averaging and rolling averages are not very large. Both methods give very similar precision and there is no clear advantage of one method over the other. Based on these figures it would make sense to choose a time period of between 10 and 30 minutes for averaging GPS points. Time intervals beyond this do not improve the precision while significantly increasing the time and distance across which dead reckoning must be applied. This lack of precision is particularly true if actual speed changes, with a constant timer setting, may be caused by field conditions.
Figure 5. Graphical representation of the maximum error from the mean for the three different stationary time trials in Bushland and the one from Ft. Collins. Rolling averages (Rlg; solid lines) and time-period averages (TP; dashed lines) are included for comparison.

Figure 6. Graphical representation of the root mean squared error from the mean point for the three different stationary time trials in Bushland and one from Ft. Collins. Rolling averages (Rlg; solid lines) and time-period averages (TP; dashed lines) are included for comparison.
The first moving test, run at the Ft. Collins site, was 2 hours in length, with the system stationary for the first hour. The system was run in the reverse direction until it reached the first reference point and then run in the forward direction for 50 minutes. The three second data are connected with a linear line (Figure 7). The 1,5 and 10 minute average data for both the radius and angle are shown. At the maximum the radius exceeds the mean of 79.6 m by 1.2 m and is less than the mean by 1.1 m. This is equal to about ±2 standard deviations of the stationary data. The data in Figure 7 shows quite good agreement between measured angular data when compared with the reference points.

Figure 7. Fort Collins center pivot system with GPS mounted near outer tower, stationary for approximately 1 hour, followed by moving in reverse direction for 27°, and then moving forward 72°. The timer setting for speed control was 100%. The calculated radius (R) and angular position (theta; θ) are shown for all of the data (3 second time interval), and for 1-minute, 5-minute, and 10-minute averages. The measured reference points are also shown for comparison.

The second moving example (Figure 8) had a similar maximum deviation of ±1.2 m from the actual radius to the GPS measured radius. This would be approximately equal to an angle difference of 0.9°. The change in θ at approximately 100 minutes is closely coupled to the change in estimated radius at the same time. An analysis of the change in θ and its change with time is better illustrated in Figure 9 and 10. The 3 second data has considerable variation from one point to another. When connecting the data points the entire figure is a series of up and down lines. This is likely due to the rapid change seen
in the GPS estimated radius caused by the limited precision of the receiver. The averaging process results with small changes in $\Delta \theta / \Delta t$ but both the 5 and 10 minute averages are quite constant. Table 4 is a summary of the mean and standard deviation of the $\Delta \theta / \Delta t$ for the 75-120 minute period for the 100 % timer and for the 20-115 minute period for the 50% timer setting. The mean angular velocity is almost the same for all averaging times. However, the standard deviation is less for both the 5 and 10 minute averaging times. It would appear that either would be a reasonable time for use in estimating the velocity for dead reckoning estimates of position corrected by GPS data. The shorter average time would be more sensitive to changes in velocity due to field conditions. The pivot’s % timer is quite accurate and could be used for dead reckoning. The GPS would then account for changes in velocity due to slippage or velocity changes due to going up or down hill.

Figure 8. Center pivot system with GPS mounted near outer tower, stationary for approximately 15 minutes, followed by moving in forward for 80°. The timer setting for speed control was 50%. The calculated radius (R) and angular position (theta; $\theta$) are shown for all of the data (3 second time interval), and for 1-minute, 5-minute, and 10-minute averages. The measured reference points are also shown for comparison.
Figure 9. Center pivot system with GPS mounted near outer tower, stationary for approximately 1 hour, followed by moving in reverse direction for 27°, and then moving forward 72°. The timer setting for speed control was 100%.

Figure 10. Center pivot system with GPS mounted near outer tower, stationary for approximately 15 minutes, followed by moving in forward for 80°. The timer setting for speed control was 50%.
Table 4. Summary statistics for the estimation of $\Delta \theta / \Delta t$ and standard deviation for the different averaging times. The reference is included for comparison.

<table>
<thead>
<tr>
<th></th>
<th>75 – 120 minutes</th>
<th>100% Timer</th>
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<tbody>
<tr>
<td></td>
<td>average $t$</td>
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</tr>
<tr>
<td></td>
<td>3 seconds</td>
<td>1 minute</td>
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<tr>
<td>$\Delta \theta / \Delta t$</td>
<td>1.57</td>
<td>1.60</td>
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<td>Std. Dev.</td>
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<tr>
<td></td>
<td>20 – 115 minutes</td>
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<tr>
<td></td>
<td>average $t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 seconds</td>
<td>1 minute</td>
</tr>
<tr>
<td>$\Delta \theta / \Delta t$</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.65</td>
<td>0.271</td>
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</table>

Additional studies with more variability in velocity would be needed to develop and test the algorithm for estimating the position of a center pivot or linear move system. The error in estimating the angular position for a center pivot would decrease as the length of the pivot increased. As was indicated, a 1.2 m variation in determining the position of a point has almost one degree error with a center pivot lateral length of 79 m. The same 1.2 m variation with a lateral length of 105 m would have less than 0.2 degree error. Precision irrigation applications with the longer laterals can be made within the expected tolerances of treatment areas.

**Conclusion**

Although differentially corrected GPS receivers report positions fairly accurately outlying position estimates are a cause for concern in precision irrigation or chemigation applications. Low-cost, differentially corrected GPS units were tested at stationary locations in Bushland Texas and in Fort Collins, Colorado. Tests on a moving center pivot were performed in Fort Collins. It was demonstrated that outlying errors from the reported GPS positions can be mitigated by averaging the reported GPS positions. Time-period averages and rolling averages were evaluated and compared and it was found that there were not large differences between the two methods. An algorithm was presented for combining averaged GPS positions from a moving irrigation system with dead reckoning for reporting real-time position. Various time periods for averaging GPS points were compared and it was found that 10 to 30 minute averages bring in the outlying errors sufficiently without having to apply dead reckoning across long times and distances. There was good agreement on the moving pivot between GPS calculated angular data when compared with measured reference points. Averaging GPS points over greater time periods to calculate angular velocity decreased the variability of velocity estimates. For estimating angular velocity averaging times of 5 to 10 minutes appeared to be adequate.
References


