# SURFACE IRRIGATION EVALUATION BASED ON ANALYTICAL INTERRELATION AMONG WATER INFILTRATION, ADVANCE, AND RECESSION

**Kamal H. Amer**

The author is **Kamal H. Amer,** Associate Professor, Department of Agricultural Engineering, Menufiya University, Shibin El-Kom, Menoufiya, Egypt; phone:2048-231-6331; fax: 202-576-9495; e-mail: kamalamer@yahoo.com.

## Abstract.

*Surface irrigation systems can be evaluated by measuring water infiltration rate, advance, and recession along irrigated field. The infiltrated water distribution depth was mathematically derived based on the three noted parameters to calculate the coefficient of variation. Evaluation of surface irrigation was done by using both mathematical and statistical analyses for water distribution depth and compared to field data. The field study was conducted at Shibin El-Kom agriculture farm in a grape field. The field is a clay loam soil with 1.28 gm/cm$^3$ bulk density and 32 mm/h saturated hydraulic conductivity and irrigated using border irrigation with strips that were 54 m long and 2.5 m wide with 0.148% slope. Inlet discharge rates of 23, 38, and 60.6 m$^3$/h were applied. Power empirical relationships were found among advance time and strip length, horizontal recession time and strip length, and infiltrated water depth and opportunity time in field situations. The infiltrated water depth was found for each inlet discharge and averaged 65.6, 70.4, and 74.5 mm at 23, 38, and 60.6 m$^3$/h, respectively. The coefficient of variation was 16.0, 12.6, and 10.3%, respectively. Excellent values uniformity coefficient (91.1%) and distribution uniformity (86.3%) were obtained, for 60.6 m$^3$/h discharge. An application efficiency of 98.3% for 23 m$^3$/h discharge due to maximal of water deficit. The highest storage efficiency (96.3%) for 60 m$^3$/h due to slight uncertain water depth. Mathematical and statistical analyses were almost exact matches of field observations. Statistical analysis could be simply used than mathematical for water profiles where maximum depth occurred in either the upstream end or the downstream.*

**Keywords.** surface irrigation, scheduling, evaluation, water distribution and efficiency.

## INTRODUCTION

In surface irrigation, irrigation water is infiltrated into root zone during conveyance and recession of water at the soil surface. The inlet irrigating stream size should be adjusted to meet the intake characteristics of the soil, the slope, and the entire area to provide a nearly uniform time for water to be infiltrated at all points along the length of the furrow, border, or basin. Three phenomena should be considered in surface irrigation design: (1) the intake characteristics of the soil; (2) the rate of advance of water front moving along the furrow or strip; (3) the rate of recession of water along the furrow or strip after water has been cutoff. The shape of water infiltrated depth depends on numerous factors, such as the variability of the soil, flow channel shape, type of irrigation (furrow versus border strip), inflow rate, irrigation hydraulics, duration of the irrigation, and slope of the field as defined by Vaziri and Wu (1972), Holzapfel et al. (1984), and Bliar and Smerdon (1988).

The general surface irrigation process may thus be considered to include three phases ( advance, vertical recession, and horizontal recession). Water advances can be defined as water traveling down slope toward the downstream end when the inflow stream is introduced at the upstream end of the plane. This phase is characterized by downfield movement of the advancing water front and continues until the water reaches the lower end of the field. After the water has advanced to the downstream end, water will continue to accumulate in the field in the vertical recession stage which is considered the storage and depletion phases in blocked-end furrow or border strip. The vertical recession continues until the depth of the surface water at the upstream end is reduced to zero. The horizontal recession phase begins when the depth of surface water at the upstream decreases to zero. This marks the initiation of the water drying or recession front. This phase continues until no water remains in the field and the irrigation is complete. The time interval during which infiltration of water into the soil can occur is bounded by the advance and recession functions and is often referred to as the infiltration opportunity time as described by Holzapfel et al. (1984) and Foroud et al. (1996). The flow pattern into root zone along furrow of surface irrigation is generally nonuniform and unsteady due to variation factors as water inflow, soil surface roughness, and infiltration rate. The water inflow is expressed in a continuity equation and an equation of motion. The equations may be solved using boundary conditions represented in infiltration and friction roughness as defined by Michael and Pandya (1971) and Cahon et al. (1995). Due to complexity of solving equations, most of cases are aimed to study individual inflow as water advance or recession as an effect on water outflow as studied by Bishop (1962) and Wu (1971). Analytical methods that used to solve continuity equation were aimed to determine approximately infiltration function, expressed in water distance run related to time, and irrigation efficiencies of surface irrigation. The derivations of infiltrated water into soil along furrow were only for two functions of surface irrigation which are advance and infiltration functions.

Warrick (1983) examined six statistical distributions of depth of water infiltrated for surface irrigation. He found uniformity coefficient (UC) as well as lower quarter distribution uniformity (DU) is related analytically to the coefficient of variation (CV) in each case. The distributions were the normal, log normal, uniform, a specialized power, beta and gamma distributions. He demonstrated that the specialized power function is exact for basin irrigation provided the surface water advance is proportional to a power of time and the intake everywhere has approached a constant value before recession. The results lend credibility to the general approximations as: (UC = 1-0.8 CV) and (DU = 1- 1.3 CV).

This research aims to study the infiltrated water distribution along border or furrow from water advance, recession, and infiltration. In this work, three functions are considered to determine the infiltrated water depth and coefficient of variation. Mathematical analysis is applied to evaluate and schedule surface irrigation. Simplicity method using statistical analysis is developed to be used for different water distribution shapes.

## THEORETICAL DEVELOPMENT

Furrow infiltration rate is an empirical power function describing the infiltration intensity as a function of opportunity time and can be expressed as:

$$I = k\ t_o^{\ n} \qquad -------- \quad (1)$$

where *I* is infiltration intensity in mm/min, *t* is an opportunity time in minute, k and n are empirical coefficients.

The cumulative infiltrated depth as a function of opportunity time can be derived by integrating the right side of

Eq. (1) respect to time and expressed as follows:

$$Z = \frac{k}{n+1}\ t_o^{\,n+1} \qquad --------- \quad (2)$$

where Z is infiltrated depth in mm, t is time in minute, and n is infiltration power coefficient which ranges from -0.2 to -0.8 for most soil types.

Water advance and recession functions together define the infiltration opportunity time along furrow length as shown in Fig. 1. The two functions can be defined as advance or recession time versus distance along the furrow and formulated in empirical equations as follows:

$$t_\ell = a\ \ell^m \qquad --------- \quad (3)$$

$$t_r = c\ \ell^x \qquad --------- \quad (4)$$

where $t_\ell$ is advance time in min, $t_r$ is recession time in min, and $\ell$ is furrow length in m, and a, c, m, and x are empirical coefficients into equations.

The infiltrated opportunity time at each furrow length point is the difference between the last time when water disappeared to the first time when water started at the same point along furrow and can be determined as follows:

$$t_o = T + t_r - t_\ell \quad ---- \quad (5)$$

where $t_o$ is infiltration opportunity time in min, T is duration time that started from water turn on and ended when the water at the upstream end disappeared in min as shown in Fig. 1. In case of vertical recession is not occurred, total time, T, is taken from water turn on to cutoff.

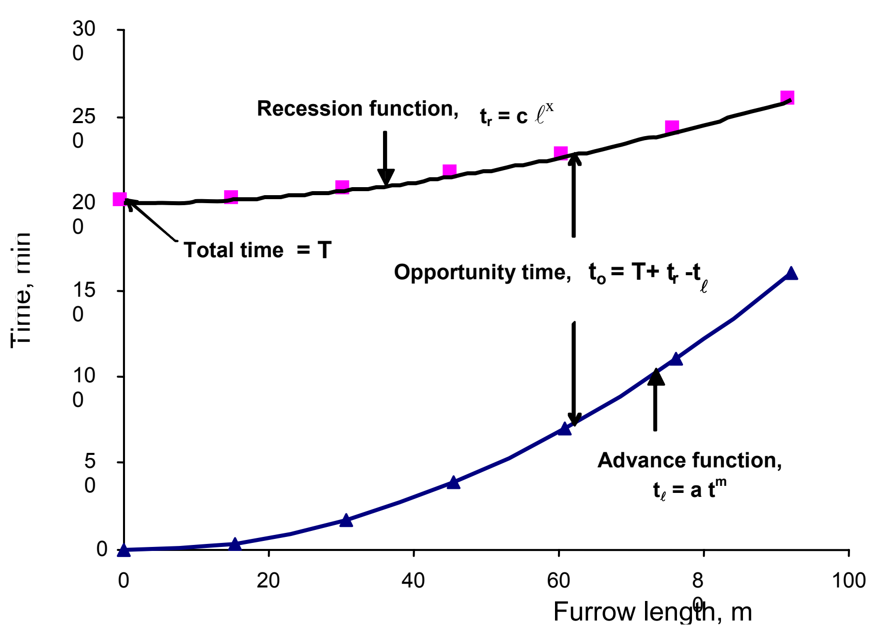


**Fig. 1: Advance and recession furrow length versus time.**

The infiltrated water depth along furrow can be formulated as follows:

$$Z = \frac{k}{n+1}(T + t_r - t_\ell)^{n+1} \quad ---- \quad (6)$$

The most preceding parameters can be employed to find the variation of infiltrated water depth (Z) from the variation of opportunity time ($t_o$) using the following statistical equation as used by Anyoji and Wu (1987) and Valiantzas (1998):

$$\frac{\sum(Z-\bar{Z})}{\bar{Z}} = (n+1)\,\frac{\sum(t_o-\bar{t}_o)}{\bar{t}_o} \qquad ------ \quad (7)$$

where $\bar{Z}$ and $\bar{t}_o$ are averages of infiltrated water depth and opportunity time, respectively, and can be formulated by integrating the right sides of Eqs. (5) and (6) respect to the furrow length after setting $\ell$ instead of t and dividing all integration by total length (L) as follows:

$$\bar{Z} = \frac{k}{n+1}\left(T + \frac{t_R}{x+1} - \frac{t_L}{m+1}\right)^{n+1} \quad ---- \quad (8)$$

$$\bar{t}_o = \left(T + \frac{t_R}{x+1} - \frac{t_L}{m+1}\right) \quad ------ \quad (9)$$

where $t_L$ and $t_R$ are total time of water advance and recession, respectively.

By applying Eq. (7) using Eqs. (5), (6), (8), and (9), the left part of Eq. (7) can be formulated after squaring and dividing both sides by total furrow length (L) as follows:

$$\frac{1}{L}\int_0^L\left(\frac{Z-\bar{Z}}{\bar{Z}}\right)^2 d\ell = \left(\frac{n+1}{\bar{t}_o}\right)^2 \frac{1}{L}\int_0^L\left((t_r - \frac{t_R}{x+1}) + (\frac{t_L}{m+1} - t_\ell)\right)^2 d\ell \quad --(10)$$

or using the following term: $CV^2 = \dfrac{(n+1)^2}{L}\displaystyle\int_0^L\left(\frac{t_o-\bar{t}_o}{\bar{t}_o}\right)^2 \cdot d\ell$

solving Eq. (10) as follows:

$$CV^2 = \left(\frac{n+1}{\bar{t}_o}\right)^2 \frac{1}{L}\int_0^L\left((t_r - \frac{t_R}{x+1})^2 + (\frac{t_L}{m+1} - t_\ell)^2 + 2\,(t_r - \frac{t_R}{x+1})(\frac{t_L}{m+1} - t_\ell)\right) d\ell$$

$$\frac{1}{L}\int_0^L\left((t_r - \frac{t_R}{x+1})^2\right) d\ell = \frac{t_R^2}{2x+1} - \frac{t_R^2}{(x+1)^2} \qquad \text{hence } t_R = c\,L^x$$

$$\frac{1}{L}\int_0^L\left((\frac{t_L}{m+1} - t_\ell)^2\right) d\ell = \frac{t_L^2}{2m+1} - \frac{t_L^2}{(m+1)^2} \qquad \text{hence } t_L = a\,L^m$$

$$\frac{2}{L}\int_0^L\left((\frac{t_L}{m+1} - t_\ell)\,(t_r - \frac{t_R}{x+1})\right) d\ell = \frac{2\,t_L\,t_R}{(m+1)(x+1)} - \frac{2\,t_L\,t_R}{m+x+1}$$

By setting $t_\ell$ in Eq. (3) and $t_r$ in Eq. (4) in Eq. (10), integrating both sides, and taking the square root of both sides, the coefficient of variation can be formulated as:

$$CV = \frac{(n+1)\cdot\sqrt{\dfrac{t_L^{\,2}}{2m+1} + \dfrac{t_R^{\,2}}{2x+1} - \dfrac{2\,t_L\,t_R}{x+m+1} - \left(\dfrac{t_L}{m+1} - \dfrac{t_R}{x+1}\right)^2}}{T + \dfrac{t_R}{x+1} - \dfrac{t_L}{m+1}} \quad --(11)$$

where CV is the coefficient of variation for water infiltrated along strip.

## MATHEMATICAL ANALYSIS OF INFILTRATED WATER DISTRIBUTION

Infiltrated water depth along furrow or strip can be profiled using Eq. (6) as shown in Fig. (2). The desired water depth, d, which soil can keep it in root zone divide the area under irrigated into three divisions which are $A_1$ represents the water stored into root zone, $A_2$ represents the water of deep seepage, and $A_3$ represents the deficit area. The infiltrated water depth, Z, can be formulated from Eq. (6) in a simple form by using binomial expansion and keeping only first two terms without significant deference as follows:

$$Z = k\sum_{r=0}^{\infty} C_r^{n+1}\, T^{n+1-r}\,(t_r - t_\ell) = k\,T^n\left(\frac{T}{n+1} + t_r - t_\ell\right) + \cdots \;---\; (12)$$

where C represents the combination and r is integral number.

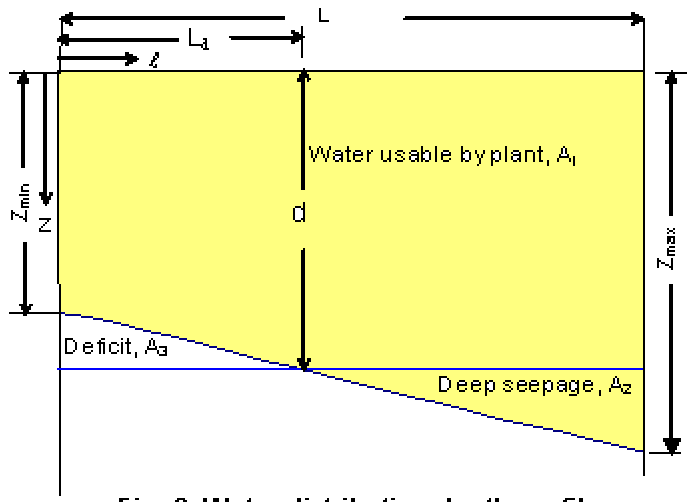


**Fig. 2: Water distribution depth profile.**

The available water depth for plant (d) is expressed as ≥ 50% soil water available in millimeters. The schedule water depth can be determined by multiply the soil root zone depth by the deference between soil volumetric water content before irrigation and after irrigation. The schedule water depth (d) segregate the irrigated area into deep seepage and deficit areas as shown in Fig. 2. Deficit area ($A_3$) which occurred at inlet furrow or strip as matched to the experimental work can be formulated as follows:

$$A_3 = d \cdot L_d - k\,T^n \int_0^{L_d}\left(\frac{T}{n+1} + t_r - t_\ell\right)\cdot d\ell \; - \;---\; (13)$$

where $L_d$ is furrow or strip length which d is occurred.

Water usable by plant area ($A_1$) can be formulated as follows:

$$A_1 = d \cdot L - A_3 \quad ----- \quad (14)$$

Deep seepage area ($A_2$) can be formulated as follows:

$$A_2 = L \cdot \bar{Z} - A_1 \quad ----- \quad (15)$$

The percentage of water deep seepage ($P_{DS}$) defined as the ratio of irrigation water drained beyond the root zone to the total water applied, the ratio of amount of water can be formulated as follows:

$$P_{DS} = \frac{A_2}{A_1 + A_2} \quad ------- \quad (16)$$

The percentage of water deficit ($P_D$) defined as the ratio of water deficit to the water needed into the root zone, can be formulated as follows:

$$P_D = \frac{A_3}{A_1 + A_3} \quad ------- \quad (17)$$

The average infiltrated depth of low quarter ($\bar{Z}_{LQ}$) that occurred at the beginning of furrow or strip water distribution can be derived as follows:

$$\bar{Z}_{LQ} = \frac{4\,k\,T^n}{L}\int_0^{L/4}\left(\frac{T}{n+1} + t_r - t_\ell\right)\cdot d\ell$$

$$\overline{Z}_{LQ} = k\,T^{n}\left(\frac{T}{n+1} + \frac{4\,t_R}{x+1}\ (0.25)^{x+1} - \frac{4\,t_L}{m+1}\ (0.25)^{m+1}\right) -- (18)$$

Water uniformity for surface irrigation profile can be determined by measuring infiltrated water along furrow or strip in systematical stations. Uniformity coefficient (UC) as a parameter that shows how water uniformly distributed along furrow can be defined as follows:

$$UC = 1 - \frac{\sum_{i=1}^{N}\left|Z - \overline{Z}\right|}{N\ \overline{Z}} \qquad -------- \quad (19)$$

where $Z_i$ is water depth measured at each station in mm, $\overline{Z}$ is mean water depth measured in all stations in mm, and N is total number of stations.

The distribution uniformity (DU) defined as the ratio of average low quarter depth of water infiltrated to the average depth of total water applied can be expressed as:

$$DU = \frac{\overline{Z}_{LQ}}{\overline{Z}} \quad ---- \ (20)$$

The application efficiency ($E_a$) defined as the ratio of amount of irrigation stored in the root zone to the total water applied, can be expressed as:

$$E_a = \frac{A_1}{A_1 + A_2} \qquad ---- \ (21)$$

The storage efficiency ($E_S$) defined as the ratio of amount of water stored to the water needed into root zone, can be expressed as:

$$E_s = \frac{A_1}{A_1 + A_3} \quad ---- \ (22)$$

## STATISTICAL ANALYSIS OF INFILTRATED WATER DISTRIBUTION

The power distribution function as developed to suit water distribution profile of surface flood irrigation can be used in statistical analysis. The infiltrated water depth (Z) along furrow or strip in dimensionless value can be expressed as (1+ α CV) where α specifies the deviation in terms of the coefficient of variation CV as shown in Fig. 3. The α-value varies from -2 to 2. The statistical analysis can be used as a standard analysis in all water distribution situations. The maximum and minimum water depth is defined as $Z_{max} = \overline{Z}$ (1+2 CV) and $Z_{min} = \overline{Z}$ (1-2 CV).

The power density function, f(α), as shown in Fig. 3 for α -value can be expressed as follows:

$$f(\alpha) = 0.375 - 0.094\,\alpha^{2} \qquad ---- \ (23)$$

where, α is a number ranges from -2 to 2 specifies the deviation of relative infiltrated water depth of terms of CV in the distribution.

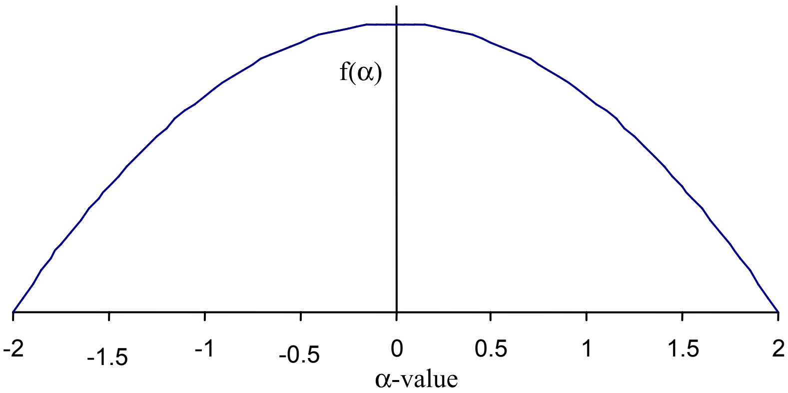

Fig. 3: Standard power distribution density function

The cumulative density function (P) of the power distribution can be expressed as:

$$P = \int_{\alpha}^{2} f(\alpha) \cdot d\alpha = 0.5 + 0.0314\,\alpha^3 - 0.375\,\alpha \qquad ---- (24)$$

The relative schedule irrigation depth in root zone ($d/\overline{Z}$) can be expressed as equals to (1+α CV) as shown in Fig. 4, the area under the frequency curve can be integrated as follows:

$$\int_{\alpha}^{2} (1+\alpha\, CV) f(\alpha) \cdot d\alpha = P + CV\,(0.6124 - 0.1535\,\alpha^2)^2 \quad ---- (25)$$

The percentage of deep seepage ($P_{DS}$) defined as the ratio of irrigation water drained beyond the root zone to the water applied can be expressed in underirrigation situation as follows:

$$P_{DS} = CV\,(0.6124 - 0.1535\,\alpha^2)^2 - \alpha\, P\, CV \quad ---- (26)$$

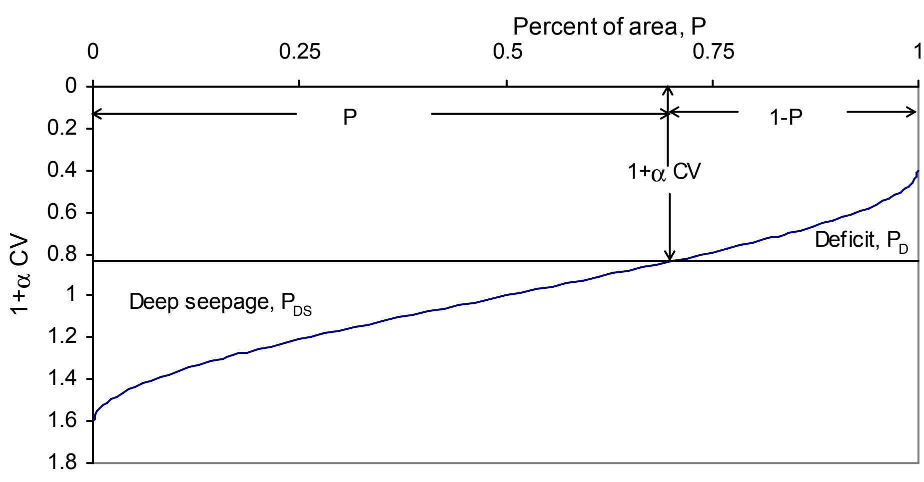

Fig. 4: Accumulative power frequency water distribution for CV=0.3 .

The percent of deficit ($P_D$) defined as the ratio of water deficit to the required water into the root zone can be expressed in underirrigation situation as follows:

$$P_D = \frac{CV\,(0.6124 - 0.1535\,\alpha^2)^2 + \alpha\, CV(1-P)}{1 + \alpha\, CV} \quad --- (27)$$

The application efficiency ($E_a$) can be expressed in power distribution as follows:

$$E_a = 1 - P_{DS} \qquad ---- (28)$$

The storage efficiency ($E_S$) can be expressed in the distribution as follows:

$$E_s = 1 - P_D \quad ---- (29)$$

The uniformity coefficient (UC) can be expressed in power distribution for water infiltrated depth which determined from Eq. 6 as follows:

$$UC = 1 - 0.86\ CV \quad ---- (30)$$

The distribution uniformity (DU) can be expressed for 100% data determined from three empirical foregoing functions as follows:

$$DU = 1 - 1.33\,CV \quad ---- (31)$$

## MATERIALS AND METHODS

*Border irrigation system was used to apply water to grape farm at the Faculty of Agriculture, Menoufiya University in shibin El-Kom area. Three inlet discharges (23, 38, 60.6 $m^3/h$) were used and had two replicates. The border strips were 54 m in length and 2.5 m in width. The shape of border strip was shown in Fig. 5. Discharge rate was adjusted using 5, and 4, and 3 inch inside diameter of PVC tube under constant head from an open channel. The water advance time was recorded for each 4.5 m strip length during irrigation time. Water was cutoff after 2 minutes from water reached the field end. The time of water cutoff ($T_{off}$) was 23, 15, and 10 min. Water recession time as a function of strip length was recorded in an empirical equation. The total flow time (T) which including the time of water advance and storage (vertical recession) was recorded from turn water on to the end of vertical recession. The infiltration rate of the classified soil was measured using double ring method before irrigation. The field slope was measured using water level tube and recorded as 0.148% down slope.*

*Soil in the study area was classified as clay loam with 1.28 g /$cm^3$ soil bulk density. Soil particle sizes for 0.3 m of soil profile were distributed as 2% coarse sand, 23.5% fine sand, 37.7% silt, and 36.80% clay. Soil particle sizes for 0.3-0.6 m of soil profile were distributed as 1.7% coarse sand, 26.3% fine sand, 32.6% silt, and 39.4% clay. The volumetric water content values were 58, 47.5, and 21.1% at saturated, field capacity, and wilting points, respectively. The Irrigation water was applied when soil water by volume was reduced to 35.3% by taken soil sample before irrigation. Soil samples were taken to 0.9 m depth along the strips in systematical stations each 4.5 m before and after irrigation. The water table at farm was more than 2.5 m.*

.

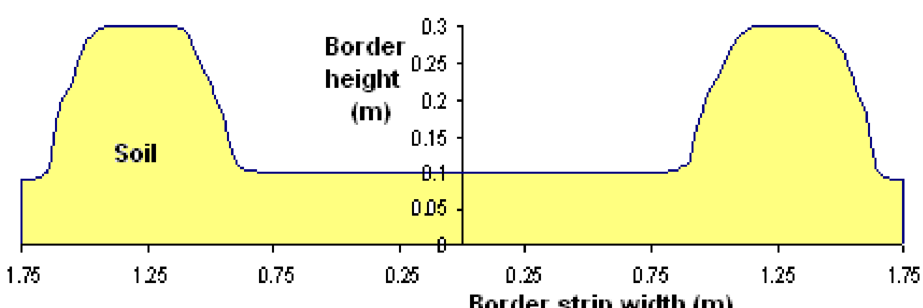


**Fig. 5: Border strip profile.**

*The irrigation schedule in grape farm was to refill water to reach the reduced water content in 0.6 m of soil depth to field capacity so irrigation interval could be determined. The schedule water depth (d) was determined as 73.2 mm for 0.6 m soil depth when soil water content by volume was averaged 35.3% as recorded before irrigation.*

## RESULTS AND DISCUSSION

An empirical equation was found in experimental site for clay loam soil to express the infiltration rate (I) in mm/min as a function of opportunity time ($t_o$) in minute as found: $I = 3.95\ t_o^{-0.423}$. The cumulative infiltrated depth (Z) in mm was integrated as: $Z = 6.846\ t_o^{0.577}$. Water advance time in minute was found as a function of strip length in meter each inlet discharge as: $t_\ell = 0.018\ \ell^{1.76}$, $t_\ell = 0.018\ \ell^{1.65}$, and $t_\ell = 0.018\ \ell^{1.53}$ for 23, 38, and 60.6 $m^3$/h inlet discharge, respectively as shown in Fig. 5. While the horizontal water recession time was functioned as: $t_r = 0.72\ \ell^{1.13}$, $t_\ell = 0.67\ \ell^{1.1}$, and $t_\ell = 0.52\ \ell^{1.12}$ for 23, 38, and 60.6 $m^3$/h, respectively. The total flow time, T, was 27, 36, 44.5 minute, respectively. The total advance time ($t_L$) was recorded as 20.2, 15, 10 min, respectively. The total horizontal recession time ($t_R$) was recorded as 65, 54, and 45.3 min, respectively. The curves in Fig. 6 showed that the greater the inlet discharge, the smaller the advance time. In the same trend, the larger the discharge the smaller the horizontal recession time. On the contrary, the smaller the inlet discharge, the larger the vertical recession time. The similar trend was earlier found by Smedema (1984) for Vertisols which referred to black cotton soils.

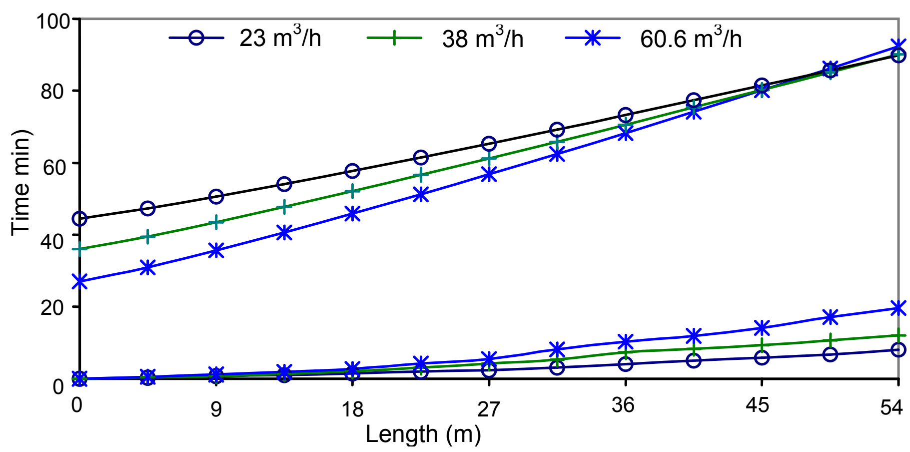


**Fig. 6: Water advance and recession versus strip length.**

Water distribution depth (Z) was determined and measured along strip for 23, 38, and 60.6 $m^3$/h as illustrated in Fig. 7. Linear regression analysis showed that highly correlation between measured and determined data, the correlation coefficient ($r^2$) was more than 0.9295 and the slope was around unity with no intercept. Measured infiltrated water depth was higher than the determined depth at the beginning of strip due to some water infiltrated from open channel and the strip end due to 0.148 % downslope. The trend of increased infiltrated depth was to increase both of strip length and inlet discharge. These results occurred due to the minimal advance time compared to recession time. As a result of the big border strip slope, the maximum irrigation depth was accumulated the end of the field. For this reason, the deviations among maximum infiltrated depths were lesser than that those among minimum infiltrated depths. The actual amount of water each irrigation was 650, 700, and 743 $m^3$/ha at 23, 38, and 60.8 $m^3$/h, respectively. The results concluded that the irrigation interval in spring time with 4 mm/day crop evapotranspiration in experimental area was 18.3 days for all inlet border strip discharge, respectively. It was 12 days in summer with 6 mm of ET.

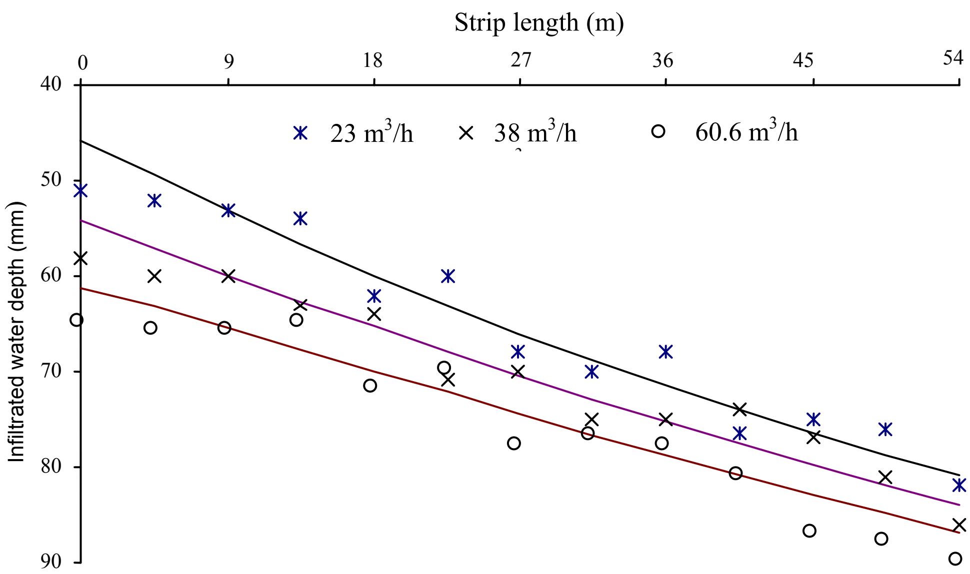


Fig. 7: Determined and field infiltrated water depth for different discharges.

Evaluation and scheduling of surface flood irrigation from the three functions of infiltration, advance rate, and recession rate were analyzed by two ways: Mathematical and Statistical Analyses. Input parameters were for 23, 38, and 60.6 $m^3$/h inlet discharge (Q) illustrated in Table 1. Advance and recession coefficients were used to determine infiltrated water depth along border strip with infiltration coefficients as k = 3.95 and n = -0.423 and total flow time (T) using Eq. 6. In addition of using total flow time (T) and three empirical functions, total advance time ($t_L$) and recession time ($t_R$) were used to determine average infiltrated depth and coefficient of variation using Eqs. 8 and 11, respectively. For the same shape of border strip and slope, the power (m) of advance function decreased when discharge increased, but the constant value (a) was almost the same. Reversely, the constant value (c) of recession function increased, but the power (x) was almost the same.

**Table 1: Input data from the experimental advance and recession functions.**

| Discharge | Advance coefficients | | Recession coefficients | | T | $t_L$ | $t_R$ |
|---|---|---|---|---|---|---|---|
| $m^3$/h | a | m | c | x | min | min | min |
| 23 | 0.018 | 1.76 | 0.52 | 1.12 | 27 | 20.2 | 65 |
| 38 | 0.018 | 1.65 | 0.67 | 1.1 | 36 | 13 | 54 |
| 60.6 | 0.018 | 1.53 | 0.72 | 1.13 | 44.5 | 8 | 45 |

## MATHEMATICAL ANALYSIS

The schedule depth (d) determined as d = 600 (0.475-0.353) = 73.2 mm water depth, was used to equivalent water depth (Z) in Eq. 6 to find out the value of $L_d$ by trial and error for 23, 38, and 60.6 $m^3$/h inlet discharge (Q) as illustrated in Table 2. Deficit area ($A_3$ in Eq. 13), water usable by plant area ($A_1$ in Eq. 14), and deep seepage area ($A_2$ in Eq. 15) were determined and used to calculate deep seepage ($P_{DS}$) and deficit ($P_D$) percentages as well as application ($E_a$ in Eq. 21) and storage ($E_s$ in Eq. 22) efficiencies. Uniformity coefficient (UC in Eq. 19) as well as distribution uniformity (DU in Eq. 20) was determined using average infiltrated depth ($\overline{Z}$ in Eq. 8) all illustrated in Table 2. The results concluded that the lower the inlet discharge, the higher the coefficient of variation. The percentage of deep seepage increased when discharge decreased and CV decreased, but water deficit percentage decreased. Uniformity coefficient (UC) as well as distribution uniformity (DU) as related to coefficient of variation (CV) increased when discharge increased and achieved acceptable value for all treatments. For 60.6 $m^3$/h inlet discharge, CU and DU achieved excellent values. Application efficiency ($E_a$) achieved a high value of 98.3% for 23 $m^3$/h discharge due to most of applied water was usable by plant, but storage efficiency ($E_s$) achieved the low value of 87.5% due to maximal of water deficit. In general, $E_a$ decreased but $E_s$ increased when schedule depth (d) is increased inbetween minimum ($Z_{min}$) and maximum ($Z_{max}$) infiltrated depths respect to increasing of coefficient of variation.

**Table 2: Output parameters of surface irrigation by mathematical analysis.**

| Q $m^3/h$ | $\overline{Z}$ mm | $L_d$ m | CV % | $P_{DS}$ % | $P_D$ % | UC % | DU % | $E_a$ % | $E_s$ % |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 65.6 | 38.87 | 16 | 1.7 | 12.6 | 86.2 | 78.5 | 98.3 | 87.4 |
| 38 | 70.4 | 31.87 | 12.6 | 3.1 | 7.4 | 89.8 | 83.1 | 96.9 | 92.6 |
| 60.6 | 74.46 | 24.44 | 10.3 | 5.1 | 3.7 | 91.2 | 86.5 | 94.9 | 96.3 |

### STATISTICAL ANALYSIS

The schedule parameter (α) determined from the following assumption:

$$1+\alpha CV=\frac{d}{\overline{Z}}$$

By arranging the foregoing equation, the parameter α will be as follows:

$$\alpha=\frac{1}{CV}\left(\frac{d}{\overline{Z}}-1\right) \quad -------- \quad (32)$$

By setting d as 73.2 mm, and $\overline{Z}$ for the three discharge treatments, the value of α was 0.777, 0.358, and -0.14 for 23, 38, 60.6 $m^3/h$ inlet discharge, respectively as illustrated in Table 3. Using α and CV from Table 3, percentages of both deep seepage ($P_{DS}$ in Eq. 26) and deficit ($P_D$ in Eq. 27) were merely determined and used to calculate both of application ($E_a$ in Eq. 28) and storage ($E_s$ in Eq. 29) efficiencies. Uniformity coefficient (UC in Eq. 30) as well as distribution uniformity (DU in Eq. 31) was determined using coefficient of variation of infiltrated water depth all data illustrated in Table 2.

**Table 3: Output parameters of surface irrigation by statistical analysis.**

| Q $m^3/h$ | $\overline{Z}$ mm | α m | CV % | $P_{DS}$ % | $P_D$ % | UC % | DU % | $E_a$ % | $E_s$ % |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 65.6 | 0.777 | 16 | 1.6 | 12.5 | 86.2 | 78.7 | 98.4 | 87.5 |
| 38 | 70.4 | 0.358 | 12.6 | 2.8 | 7 | 89.2 | 83.2 | 97.2 | 93 |
| 60.6 | 74.46 | -0.14 | 10.3 | 5.2 | 3.2 | 91.1 | 86.3 | 94.8 | 96.8 |

Both mathematical and statistical analyses were achieved almost the same results of predicting $P_{DS}$, $P_D$, UC, DU, $E_a$, and $E_s$ as shown in Tables 2 and 3. Statistical analysis achieved abbreviation in output calculation because of simplicity of determining α-value (using Eq. 32) directly from the schedule depth (d) and coefficient of variation (CV). However, mathematical analysis achieved the same output data in complexity calculation as a result of finding indirectly the length ($L_d$) where the schedule depth (d) was occurred along border strip or furrow by trial and error method. Also statistical method was typically applied for both of water distribution shapes where maximum depth occurred at up-field or down-field. In contrast, mathematical analysis should be analyzed each shape due to switching the places of deep seepage and water deficit.

## Conclusion

Surface irrigation systems is widely used to irrigate most of the traditional crops in northern of Egypt where most of the old irrigated lands located. It can be managed and evaluated by using three functions for the field situation, these are water infiltration, advance rate, and recession rate. Infiltrated water distribution depth was mathematically derived based on the three functions. Evaluation of surface irrigation was done by using both mathematical and statistical analyses for infiltrated water distribution profile and compared to field data. A study was conducted on a grape grown at the Agriculture College farm at Shibin El-Kom area on a clay loam soil with 1.28 gm/cm$^3$ bulk density and 32 mm/h saturated hydraulic conductivity. Border irrigation was used to feed water into strips with 0.148% downslope, 54 m long and 2.5 m width. Inlet discharge rates of 23, 38, and 60.6 m$^3$/h were applied. Power empirical relationships were found among advance time and strip length, horizontal recession time and strip length, and infiltrated water depth and opportunity time in field situations. The results were found as follows:

- Infiltrated water depth along strip was found and averaged as 65.6, 70.4, and 74.5 mm at 23, 38, and 60.6 m$^3$/h, respectively.
- The coefficient of variation was 16, 12.6, and 10.3% at 23, 38, and 60.6 m$^3$/h, respectively.
- The amount of water each irrigation was 650, 700, and 743 m$^3$/ha at 23, 38, and 60.6 m$^3$/h, respectively.
- Uniformity coefficient as well as distribution uniformity increased when inlet discharge increased but acceptable values achieved for all discharge treatments, although the UC for 60.6 m$^3$/h was the highest.
- Application efficiency achieved a high value of 98.3% for 23 m$^3$/h discharge due to maximal of water deficit, but storage efficiency achieved the high value of 96.3% for 60.6 m$^3$/h due to minimal of water deficit.
- Mathematical and statistical analyses were achieved almost the same results of predicting output parameters.
- Statistical analysis achieved an abbreviation in output calculations than mathematical analysis due to simplicity of application in evaluating and scheduling irrigation systems.

## REFERENCES

Amer, K. H. 2005. Trickle irrigation schedules and evaluation. Misr J. Ag. Eng., 22(3): 899-922.

Anyoji, H. and I.P. Wu (1987). Statistical approach for drip lateral design. Trans. of the ASAE 30(1): 187-192.

Bishop, A. A. 1962. Relation of intake rate to length of run in surface irrigation. Trans. of the ASCE 127 part III.

Blair, A. W. and E. T. Smerdon. 1988. Unimodal surface irrigation efficiency. J. of Irrig. And Drain. Div., ASCE, 114(1), 156-167.

Cahoon, J. E., P. Mandel, and D. E. Eisenhauer. 1995. Management recommendations for sloping blocked-end furrow irrigation. Applied Engineering in Agriculture, ASAE, Vol. 11(4): 527-533.

Holizapfel, E. A., M. A. Marino, and J. Chavez-Morales. 1984. Comparison and selection of furrow irrigation models. Agricultural water management, Elsevier Sci. Pub. 9: 105-125.

Foroud, N., E. St. George, and T. Entz. 1996. Determination of infiltration rate from border irrigation advance and recession trajectories. Agricultural water management, Elsevier Sci. Pub. 30: 133-142.

Michael, A. M., and A. C. Pandya. 1971. Hydraulic resistance relationship in irrigation borders. J. Agric. Engng. Res., 13(1): 72-80.

Smedema, L. K. 1984. Furrow irrigation design for Vertisols. Agricultural water management, Elsevier Sci. Pub. 9: 211-218.

Valiantzas, J.D. (1998). Analytical approach for direct drip lateral hydraulic calculation. J. Irrig. And Drain. Engng., ASCE 124(6): 300-305

Vaziri, C. M. and I. P. Wu. 1972. Volume balance analysis to determine water infiltration and efficiency in Hawaiian sugarcane furrows. Hawaiian Sugar Planters Association, 58(20). 283-292.

Warrick, A.W. 1983. Interrelationship of irrigation uniformity terms. J. Irrig. & Drain. Eng., ASCE 109(3): 317-331.

Wu I.P. 1971. Overland flow hydrograph analysis to determine infiltration function. Trans. of the ASAE. 14(2): 294-300.